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Gravitational couplings, orientifolds and M-planes

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Abstract: We examine string-theory orientifold planes of various types including the Sp and SO-odd planes, and deduce the gravitational Chern-Simons couplings on their world-volumes. Consistency checks are carried out in different spacetime dimensions using various dualities, including those relating string theory with F-theory and M-theory. It is shown that when an orientifold 3-plane crosses a 5-brane, the jump in the charge is accompanied by a corresponding change in the gravitational couplings.

Keywords: M-Theory, D-branes, String Duality, Anomalies in Field and String Theories.

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Contents

1. Introduction

Topological or Chern-Simons couplings on Dirichlet branes have played an important role in string theory and the understanding of dualities. These couplings involve two kinds of fields: fields arising in the bulk spacetime theory (from the RR or NS-NS sectors) and fields coming from the brane worldvolume theory.

In particular, there are gravitational couplings of this kind involving only bulk fields, namely the spacetime metric and various m-form potentials. Schematically, they are of three types:

$$\int C \wedge \mathrm{tr}(R \wedge R)\,, \qquad \int C \wedge (\mathrm{tr}(R \wedge R))^2\,, \qquad \int C \wedge \mathrm{tr}(R \wedge R \wedge R \wedge R)\,. \quad (1.1)$$

In each expression, C is some m-form gauge potential where m is such that the term has the correct rank to be integrated over the relevant brane world-volume. The second and third terms involve 8-forms in the curvature, hence they arise only on p-branes for $p \geq 7$. We will refer to these as "8-form terms". The first term occurs on all D p-branes for $3 \leq p \leq 9$, and will be referred to as a "4-form" term. The presence of such terms was first predicted (in a special case) in ref. [1] using duality. The general form was worked out using an anomaly inflow argument in ref. [2], and extended to the case of a twisted normal bundle in ref. [3].

The above terms do not involve world-volume fields of the D-brane, so they can be thought of as interactions among bulk fields that are localized on the brane. From this perspective, there is no reason why orientifold planes cannot have similar interactions residing on them. Indeed, it was first observed in ref. [4] that orientifold planes indeed have gravitational 4-form and 8-form world-volume couplings, whose coefficients were determined using heterotic–type-I duality. The presence of such interactions was tested in ref. [4, 5] using dualities involving M and F theory.

In recent times, the presence of gravitational couplings on both branes and planes has been confirmed by explicit perturbative string computations. In particular, as far as $\mathcal{O}$-plane couplings are concerned, in ref. [6] the gravitational 4-form couplings predicted in ref. [4] were confirmed by computing appropriate scattering amplitudes. Refs. [7, 8] performed more general computations, thereby determining the 8-form couplings as well, and correcting a numerical error in ref. [4]. They also noted that the $\mathcal{O}$-plane gravitational couplings are summarized in the Hirzebruch polynomial, just as the D-brane gravitational couplings are summarized in the A-roof genus. The complete result including the extension to the case of a nontrivial normal bundle is to be found in ref. [7], while related further work was carried out in refs. [9, 10].

The orientifold planes considered in the above works are the ones that give rise to D_n gauge theories when branes coincide with them. They always carry a negative amount of D-brane charge and are usually called $\mathcal{O}p^-$ planes.[1] The purpose of this note is to examine other kinds of orientifold planes from the perspective of world-volume gravitational couplings. In particular, we will focus on the kind which give rise to C_n or Sp-type gauge theories when branes coincide with them. These always carry a positive charge and will be denoted $\mathcal{O}p^+$ in what follows.

At least two other types of orientifold planes exist, but they have been observed only in a few specific (low) dimensions. One of them, which produces $\mathrm{SO}(2n+1)$ type gauge groups, consists of an $\mathcal{O}p^-$ plane with a single D-brane stuck on it, and will be denoted $\tilde{\mathcal{O}}p$. The other type of plane is somewhat mysterious and gives rise to Sp-type gauge groups, but differs from the $\mathcal{O}p^+$ plane mentioned above. Following ref. [12], we will denote it $\hat{\mathcal{O}}p$.

The D p-brane charge[2] of an $\mathcal{O}p^-$ plane is -2^{p-5}, while for the $\mathcal{O}p^+$ plane it is 2^{p-5}. For the $\tilde{\mathcal{O}}p$ plane the charge is $-2^{p-5}+1/2$ in the dimensions where it exists, while for the $\hat{\mathcal{O}}p$ plane the charge is known only in a few specific dimensions, as we will discuss below. The $\tilde{\mathcal{O}}p$ do not have simple transformation properties under T-duality, while the $\mathcal{O}p^-$ and $\mathcal{O}p^+$ planes simply double each time we compactify on a circle and T-dualize. We will see in one case that assuming this also for the $\hat{\mathcal{O}}$-plane leads to a consistent result.

2. Gravitational couplings on $\mathcal{O}^-$ planes

The general formulae obtained in ref. [4] can be summarised as follows. An orientifold p-plane of type $\mathcal{O}p^-$, for $3 \leq p \leq 9$, has a gravitational 4-form coupling on its world-

[1] Unfortunately, they are also sometimes called $\mathcal{O}^+$ planes [11]. We will not use this notation.
[2] These charges are in units where a mirror pair of D-branes has charge $+1$.

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volume given by

$$\mathcal{O}p^- : \quad -\frac{2^{p-9}}{3}\int C^{RR}_{p-3}\wedge\frac{\mathrm{tr}\,R\wedge R}{16\pi^2} = -\frac{2^{p-9}}{6}\int C^{RR}_{p-3}\wedge p_1\,, \tag{2.1}$$

where p_1, the first Pontryagin class of the manifold, is given by

$$p_1(R) = \frac{1}{8\pi^2}\,\mathrm{tr}\,R\wedge R \tag{2.2}$$

and C^{RR}_{p-3} is the $(p-3)$-form Ramond-Ramond gauge potential. For comparison, a single Dp-brane carries the coupling

$$Dp : \quad -\frac{1}{48}\int C^{RR}_{p-3}\wedge p_1\,. \tag{2.3}$$

This formula is unambiguous in the absence of orientifold planes. However, when an orientifold plane is present, D-branes can only move around in mirror pairs. In this case, a single D-brane of a mirror pair carries the coupling

$$-\frac{1}{96}\int C^{RR}_{p-3}\wedge p_1\,. \tag{2.4}$$

In particular, in situations where "half" a D-brane is considered to be stuck at the orientifold, this object carries the term in eq. (2.4).

For $7\le p\le 9$ there are also gravitational 8-form couplings [4]. An $\mathcal{O}p^-$ plane carries the coupling[3]

$$\mathcal{O}p^- : \quad 2^{p-9}\int C^{RR}_{p-7}\wedge\left(\frac{1}{640}(p_1)^2-\frac{7}{1440}p_2\right), \tag{2.5}$$

where

$$p_2 = \frac{1}{(2\pi)^4}\left(-\frac{1}{4}\,\mathrm{tr}\,R^4+\frac{1}{8}\,(\mathrm{tr}\,R^2)^2\right) \tag{2.6}$$

For comparison, a single independent D p-brane carries the 8-form coupling:

$$Dp : \quad \frac{1}{320}\int C^{RR}_{p-7}\wedge\left(\frac{1}{8}\,(p_1)^2-\frac{1}{9}\,p_2\right). \tag{2.7}$$

The formulae above for gravitational couplings on D-branes and $\mathcal{O}^-$-planes can be summarised as follows [2, 7, 8]:

$$\begin{aligned} Dp : &\quad \int\sum_i C^{RR}_i\wedge\sqrt{\hat{A}(R)} \\ \mathcal{O}p^- : &\quad -2^{p-5}\int\sum_i C^{RR}_i\wedge\sqrt{\hat{\mathcal{L}}(R/4)}\,, \end{aligned} \tag{2.8}$$

[3]The correct answer is to be found in the latest hep-th version of ref. [4], superseding the published version.

where $\sum_i C_i^{RR}$ is the formal sum of all RR gauge potentials in the corresponding string theory (type IIA or IIB), and $\hat{A}$ and $\hat{\mathcal{L}}$ are the A-roof genus and Hirzebruch polynomials respectively, given by:

$$\begin{aligned}\hat{A}(R) &\equiv 1 - \frac{1}{24}p_1 + \frac{7}{5760}(p_1)^2 - \frac{1}{1440}p_2 + \cdots \\ \hat{\mathcal{L}}(R/4) &\equiv 1 + \frac{1}{48}p_1 - \frac{1}{11520}(p_1)^2 + \frac{7}{11520}p_2 + \cdots \end{aligned} \tag{2.9}$$

Only the forms of the correct rank to be integrated over the brane or plane worldvolume are retained in eq. (2.8). Moreover, this formula is valid only when the normal bundle to the world-volume is trivial. Otherwise, we must make the replacements:

$$\begin{aligned}\hat{A}(R) &\to \frac{\hat{A}(R_T)}{\hat{A}(R_N)} \\ \hat{\mathcal{L}}(R/4) &\to \frac{\hat{\mathcal{L}}(R_T/4)}{\hat{\mathcal{L}}(R_N/4)}\end{aligned} \tag{2.10}$$

where R_T and R_N are the curvature 2-forms of the tangent and normal bundles respectively.

3. Gravitational couplings on other $\mathcal{O}$ planes

Let us now consider the other kinds of orientifold planes. Recall that the above results were originally derived in ref. [4] by appealing to heterotic/type-I duality and the low-energy effective action of the heterotic string. The anomalous Bianchi identity satisfied by the heterotic string 3-form $H = dB$ can be dualized to a coupling

$$-\frac{1}{2}\int B_6 \wedge p_1(R) \tag{3.1}$$

in the effective action of the heterotic string, where B_6 is the dual potential to B, in other words $dB_6 = {}^*dB$ where * is the Hodge dual. Dualizing to type I, B_6 becomes an RR 6-form potential. Requiring that this term comes from one $\mathcal{O}9^-$ plane and 16 D9-branes, and using the results of ref. [2] for gravitational couplings on D-branes, one finds the result for the coupling on an $\mathcal{O}9^-$ plane. Finally, the fact that $\mathcal{O}^-$ planes split into two each time we T-dualize gives the result for all $\mathcal{O}^-$ planes.

The same procedure works for 8-form couplings, except that this time the relevant terms in the effective action come not from the anomalous Bianchi identity but from the Green-Schwarz cancellation mechanism.

If we want to deduce the gravitational couplings on Sp-type orientifold planes $\mathcal{O}p^+$, we need to produce a string vacuum containing these planes, and where the relevant terms in the effective action are known. A convenient example is the 8-dimensional compactification discussed in ref. [11], dual to a toroidal compactification

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without vector structure. This has three $\mathcal{O}7^-$ planes and one $\mathcal{O}7^+$ plane, and 8 pairs of D7-branes. It is easy to see that the terms that were used in ref. [4] to find the gravitational couplings on $\mathcal{O}^-$ planes are present in this 8-dimensional model with the same coefficients as before. These terms arise as higher derivative (or α') corrections in the tree-level effective action. The 8-dimensional model in question can be dualized to a CHL [13] type background for the heterotic string. This corresponds to a Z_2 orbifold without a twisted sector, hence the tree-level couplings of the B-field and gravitons (which are all invariant under Z_2) are unchanged.

With this result it is straightforward to find the 4-form and 8-form couplings on the $\mathcal{O}7^+$ plane. In the present example, the 4-form coupling on three $\mathcal{O}7^-$ planes, a $\mathcal{O}7^+$ plane and 8 pairs of D7-branes must add up to the term in eq. (3.1). Moreover, an $\mathcal{O}^+$ plane also splits into two upon T-dualizing, like the $\mathcal{O}^-$ plane [11]. It follows that an $\mathcal{O}p^+$ plane (which exists for all $p \leq 8$) carries the gravitational 4-form coupling:

$$\mathcal{O}p^+ : \quad -\frac{5.2^{p-8}}{12} \int C^{RR}_{p-3} \wedge p_1 \,. \tag{3.2}$$

We will check this result with various dualities below.

Another type of orientifold plane that exists for $p \leq 8$ will be denoted $\tilde{\mathcal{O}}p$. This is the plane that gives rise to gauge symmetries of type $\mathrm{SO}(2n+1)$. It consists of a single D-brane (sometimes called a "half" D-brane) stuck on an $\mathcal{O}^-$ plane. Hence its gravitational 4-form coupling is

$$\tilde{\mathcal{O}}p : \quad -\frac{2^{p-5}+1}{96} \int C^{RR}_{p-3} \wedge p_1 \,. \tag{3.3}$$

Next let us turn to 8-form couplings. Here, considerations analogous to the ones discussed above lead to the expressions:

$$\begin{aligned} \mathcal{O}p^+ : &\quad 2^{p-8} \int C^{RR}_{p-7} \wedge \left(\frac{9}{1280}(p_1)^2 - \frac{23}{2880} p_2 \right), \\ \tilde{\mathcal{O}}p : &\quad \int C^{RR}_{p-7} \wedge \left(\frac{2^{p-6}+1}{5120}(p_1)^2 - \frac{7.2^{p-7}+1}{5760} p_2 \right). \end{aligned} \tag{3.4}$$

It is straightforward to check, using the above expressions, that the 9-dimensional vacuum containing three $\mathcal{O}7^-$ planes, one $\mathcal{O}7^+$ plane and 8 pairs of D7-branes, has the total 8-form term:

$$\int \phi^{RR} \wedge \left(\frac{1}{128}(p_1)^2 - \frac{1}{96} p_2 \right), \tag{3.5}$$

which is the dimensional reduction to 8 dimensions of the gravitational part of the Green-Schwarz anomaly-cancelling term (here, ϕ^{RR} is the Ramond-Ramond scalar of type IIB string theory).

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The results above for the gravitational couplings on an $\mathcal{O}p^+$-plane can be summarised in the formula:

$$\mathcal{O}p^+ : \quad 2^{p-5}\int \sum_i C_i^{RR} \wedge \left(2\sqrt{\frac{\hat{A}(R_T)}{\hat{A}(R_N)}} - \sqrt{\frac{\hat{\mathcal{L}}(R_T/4)}{\hat{\mathcal{L}}(R_N/4)}} \right). \tag{3.6}$$

Here, in addition to the tangent bundle contribution which we derived above, we have conjectured the contribution of the normal bundle when this is nontrivial.

4. Consistency with dualities

The above formulae for gravitational couplings can be checked with various dualities in different dimensions. These checks are not strictly independent of the considerations that gave rise to eq. (3.6), yet it is useful to carry them out explicitly to reassure oneself that a sensible picture emerges in every dimension.

The first check arises in 8 dimensions, where the vacuum with three $\mathcal{O}7^-$ planes and one $\mathcal{O}7^+$ plane was argued [11] to be dual to a special F-theory compactification with a "frozen" D_8 singularity. Macroscopically, this is identical to a conventional D_8 singularity which can split into 10 generic singular fibres. The 4-form gravitational couplings on a 7-brane or 7-plane are $\mathrm{SL}(2,\mathbb{Z})$ invariant, hence we expect that the world-volume gravitational couplings of the frozen D_8 singularity are equal to 10 times the worldvolume couplings of a D7-brane. From eq. (3.2), with $p = 7$, we find that the coefficient of the coupling is $-5/24$, which is 10 times the coupling of a single D7-brane, $-1/48$, as expected.

In 7 dimensions, the $\mathcal{O}6^+$ plane has similarly been related to M-theory with a "frozen" D_4 singularity [14]. For us, the key feature of this otherwise mysterious singularity is that it is geometrically the same as an $\mathcal{O}6^-$ plane with four D6-branes on top of it. In this dimension, the gravitational coupling on an $\mathcal{O}6^-$ plane and on a D6-brane was re-derived from M-theory in ref. [5]. This derivation proceeds by starting with the well-known "one-loop" gravitational coupling $\int C_3 \wedge I_8(R)$ in eleven-dimensional M-theory, where $I_8(R) = \frac{1}{24}\left(p_2 - \frac{1}{4}(p_1)^2\right)$. Integrating this over the 4-dimensional manifold representing the transverse space to a 6-brane or 6-plane, we get the induced gravitational coupling on the corresponding object.

Both for a 2-centre euclidean Taub-NUT space, which is the transverse space to a D6-brane, and for the Atiyah-Hitchin space, transverse to an $\mathcal{O}6^-$-plane, it is known that $\int p_1 = 2$. Thus, by the above arguments, for the space transverse to an $\mathcal{O}6^+$-plane we must have $\int p_1 = 10$. This gives rise to the induced gravitational coupling

$$\mathcal{O}6^+ : \qquad -\frac{5}{48}\int C_3 \wedge p_1 \,, \tag{4.1}$$

which agrees with eq. (3.2) for $p = 6$.

In 5 dimensions, we can use the beautiful results of Hori [15] relating various types of $\mathcal{O}4$-planes to M-theory orientifolds [16, 17, 18]. The $\mathcal{O}4^-$ plane is relatively straightforward, as it corresponds in M-theory to an orientifold 5-plane (which we will refer to as an $\mathcal{OM}$ 5-plane). The $\mathcal{O}4^+$ plane is, according to ref. [15], an $\mathcal{OM}$-plane with a pair of M-theory 5-branes stuck to it. Now, neither the M5-brane nor the $\mathcal{OM}$ 5-plane carry gravitational couplings [19]. However, being chiral objects, they carry an analogue of the $\mathrm{SL}(2,\mathbb{Z})$ anomaly of ref. [20]. It was argued in ref. [19], by analogy with a similar mechanism discussed in ref. [21], that after compactifying on a circle the chirality of these objects gives rise to the appropriate gravitational couplings on the world-volumes of D4-branes and $\mathcal{O}4^-$-planes. It follows that the gravitational coupling on the $\mathcal{O}4^+$ plane is equal to that induced by an $\mathcal{OM}$ 5-plane and a pair of $M5$-branes, which in turn is equal to the sum of gravitational couplings on an $\mathcal{O}4^-$-plane and a pair of D4-branes. This leads to the coupling $-\frac{5}{192}\int A^{RR}\wedge p_1$, in agreement with eq. (3.2) for $p=4$.

The case of $\tilde{\mathcal{O}}$ 4-planes, which give rise to $\mathrm{SO}(2n+1)$ gauge groups when branes coincide with them, is rather different. According to ref. [15], these arise in M-theory by combining the orientifolding operation in five directions with a half-shift along the M-direction x^{10}. This action has no fixed points, hence from the M-theory point of view there is no orientifold plane at all. However, in the type IIA limit (as the radius of the x^{10} direction goes to zero) the half-shift is not geometrically visible and we seem to have an orientifold 4-plane. In fact the half-shift is now realized as half a unit of Ramond-Ramond flux $\int A^{RR}$ along any curve that passes through the orientifold plane.

While it would be desirable to propose an M-theoretic origin for the gravitational couplings on the $\tilde{\mathcal{O}}$ 4-planes (analogous to chirality, discussed above for the $\mathcal{O}4^{\pm}$ planes), this does not seem to be straightforward, precisely because in the M-theory limit this 4-plane is not an orientifold at all. However, it is interesting to note that this orientifold 4-plane is closely related to an uncharged orientifold 8-plane that has been discussed in the literature [22]. Consider the compactification of M-theory to 9 spacetime dimensions on a Klein bottle, defined as the quotient of a 2-torus labelled by x^9, x^{10} by the action $x^9\to -x^9, x^{10}\to x^{10}+\pi R^{10}$. From the M-theory point of view this has no fixed points, but in the string theory limit one finds two fixed planes at $x^9=0,\pi R^9$, with the half-shift in x^{10} being realized as half a unit of A^{RR}-flux through each fixed plane. These two planes are neutral and the vacuum is therefore "braneless". According to ref. [11], this model is dual to another braneless orientifold, without RR flux, containing one $\mathcal{O}8^-$ and one $\mathcal{O}8^+$ plane. Hence the gravitational coupling on this exotic $\mathcal{O}$ 8-plane is the average of the couplings on the $\mathcal{O}8^-$ and $\mathcal{O}8^+$ planes, or:

$$\text{neutral } \mathcal{O}8: \quad -\frac{1}{4}\int C_5^{RR}\wedge p_1\,. \tag{4.2}$$

While the neutral 8-planes carry RR flux and are therefore difficult to study in perturbation theory (like the orbifold planes in ref. [23]), the dual vacuum containing an $\mathcal{O}8^-$ and an $\mathcal{O}8^+$ plane can be described very precisely as a perturbation series [11].

One can analogously realize $\tilde{\mathcal{O}}$ 4-planes in a braneless vacuum by compactifying M-theory to 5 spacetime dimensions on a "Klein six-bottle", defined as the quotient of the 6-torus $x^5, \ldots, x^{10}$ by the action $(x^5, \ldots, x^9) \to -(x^5, \ldots, x^9)$, $x^{10} \to x^{10} + \pi R^{10}$. This time, in the string theory limit there are 32 neutral orientifold planes, each carrying half a unit of RR flux. According to ref. [15], this vacuum is dual to the string theory vacuum discussed in ref. [18] where there are 32 $\tilde{\mathcal{O}}4$-planes, each consisting of "half" a D4-brane stuck to an $\mathcal{O}4^-$-plane. It follows that, like the neutral $\mathcal{O}8$-planes, the neutral $\tilde{\mathcal{O}}4$-planes carry gravitational couplings

$$\tilde{\mathcal{O}}4: \quad -\frac{1}{64}\int A^{RR} \wedge p_1 \tag{4.3}$$

in agreement with eq. (3.3).

We can also examine the exotic Sp-type 4-plane (which we denoted $\hat{\mathcal{O}}4^+$). This is supposed to arise [15] as an M5-brane wrapped on the cylinder over which the $\tilde{\mathcal{O}}4$-plane is smeared. In the string theory limit this brane will descend to half a D4-brane, hence the coupling on this 4-plane is:

$$\hat{\mathcal{O}}4: \quad -\frac{5}{192}\int A^{RR} \wedge p_1\,, \tag{4.4}$$

which is the same as we found for the $\mathcal{O}4^+$-plane. Thus it appears that both types of Sp-planes have the same gravitational coupling, as well as the same 4-brane charge.

Finally we come to 4 spacetime dimensions, or orientifold 3-planes. Here we find the pleasant result, from eqs. (3.2), (3.3) and (4.4) (in the last case we assume that the $\hat{\mathcal{O}}4$-plane also doubles on compactification and T-duality) that three different types of orientifold 3-planes have the same gravitational coupling:

$$\mathcal{O}3^+, \tilde{\mathcal{O}}3, \hat{\mathcal{O}}3: \quad -\frac{5}{384}\int \tilde{\phi} \wedge p_1 \tag{4.5}$$

consistent with the fact [24] that these three types of $\mathcal{O}3$-planes are permuted into each other by $\mathrm{SL}(2,\mathbb{Z})$ S-duality.

5. Intersecting orientifolds and branes

An interesting application of the above considerations can be found in a system where an orientifold plane intersects a brane. Here we will consider the case when an orientifold 3-plane intersects an NS 5-brane in type IIB string theory. The worldvolume directions are (x^1, x^2, x^3) for the 3-plane and $(x^1, x^2, x^4, x^5, x^6)$ for the 5-brane, so they intersect over a 2-brane in the (x^1, x^2) directions. This is a supersymmetric configuration.

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As shown in ref. [24], an $\mathcal{O}3^-$ plane intersecting an NS5-brane turns into an $\mathcal{O}3^+$ plane on the other side.[4] The latter is the standard Sp-type plane that we have discussed in previous sections. The particular property of $\mathcal{O}3$-planes is that the space transverse to them contains a 2-cycle (an RP^2) on which the fluxes $\theta_{NS} = \int B_{NS}$ and $\theta_{RR} = \int B_{RR}$ can take two discrete values, 0 and 1/2. The effect of crossing an NS5-brane is that

$$(\theta_{RR}, \theta_{NS}) \to \left(\theta_{RR}, \theta_{NS} + \frac{1}{2}\right) \tag{5.1}$$

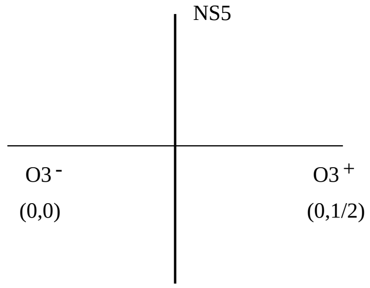

Figure 1: $\mathcal{O}3$-plane with $\theta_{RR} = 0$ intersecting an NS5-brane.

where the θ-values are understood to be mod 1. Hence in fig. 1, the plane on the left is of type $(0, 0)$ and the one on the right is $(0, 1/2)$.

We will now see that the world-volume gravitational couplings on the orientifold planes jump across the NS5-brane. Consider an RP^5 enclosing the $\mathcal{O}3^-$ on the left. It contains the RP^2 cycle on which we evaluate $(\theta_{RR}, \theta_{NS})$ to be $(0, 0)$. Now move this RP^5 towards the right. It intersects the NS5-brane in an RP^2, enclosing the 2-brane intersection region denoted in the figure by a point. Recall that the 3-brane charge of the orientifold planes jumps across the NS5-brane, from $-1/4$ to $+1/4$. It is amusing to note that this is very much like branes ending on branes [26], in that a net 3-brane charge is deposited on the 5-brane. This is usually interpreted as the fact that the intersection region acts like a magnetic source in the 5-brane world-volume, carrying the net charge deposited on the 5-brane.

In the present case, this means that the (twisted) world-volume U(1) gauge field strength on the NS5-brane must be excited, with

$$\int_{RP^2} F = \frac{1}{2} \tag{5.2}$$

(in the covering space, the charges on the planes would be doubled and we would have $\int F = 1$). Now let us consider the world-volume couplings on the NS5-brane:

$$\int F \wedge D^+ - \frac{1}{48} \int \tilde{\phi}\, F \wedge p_1(R)\,. \tag{5.3}$$

In the presence of the world-volume flux through RP^2, these give rise to co-dimension 2 couplings:

$$\frac{1}{2} \int D^+ - \frac{1}{96} \int \tilde{\phi}\, p_1(R)\,. \tag{5.4}$$

From the geometry of the problem, it follows that the world-volume couplings on the $\mathcal{O}3^-$ plane on the left, added to the above terms, must equal the world-volume

[4] This phenomenon was first discovered in ref. [25] for $\mathcal{O}4$-planes.

couplings of the $\mathcal{O}3^+$ plane on the right. For the $\mathcal{O}3^-$ plane, we have the couplings

$$-\frac{1}{4}\int D^+ - \frac{1}{384}\int \tilde{\phi}\, p_1(R)\,. \tag{5.5}$$

Adding this to eq. (5.4), we find the result

$$\frac{1}{4}\int D^+ - \frac{5}{384}\int \tilde{\phi}\, p_1(R)\,. \tag{5.6}$$

This exhibits the fact that the $\mathcal{O}3^+$ plane has 3-brane charge $+1/4$ as expected, and also a gravitational 4-form coupling with coefficient $-5/384$ as predicted in previous sections.

Note that this jump in the world-volume coupling is not a simple consequence of crossing the NS5-brane, as it only takes place when a world-volume flux is excited. For a different example, consider the $\mathcal{O}3$-plane of type $(1/2, 0)$, which is the SO-odd orientifold plane. When this intersects an NS5-brane, the plane on the right is the $\mathcal{O}3$-plane of type $(1/2, 1/2)$, which is the "exotic" Sp plane.

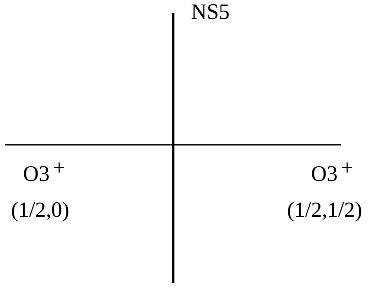

Figure 2: $\mathcal{O}3$-plane with $\theta_{RR} = 1/2$ intersecting an NS5-brane.

In this case, the orientifold planes on the left and right of the figure both have 3-brane charge $+1/4$. Hence, even though the value of θ_{NS} jumps as before, there is no jump in the 3-brane charge. It follows that no net charge is deposited on the NS5-brane, hence no world-volume flux is excited. This in turn tells us that the gravitational coupling does not jump in this case, in accordance with the results of the previous section.

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